Ocean Modelling xxx (2011) xxx–xxx

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

Short communication

On the application of no-slip lateral boundary conditions to 'coarsely' resolved ocean models

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ARTICLE INFO

Article history:
Received 15 February 2011
Received in revised form 4 May 2011
Accepted 8 May 2011
Available online xxxx

Keywords:
Ocean modeling
Boundary conditions

ABSTRACT

The problem of no-slip boundary conditions as they apply to ocean models is revisited. It is argued that the setting is consistent with classical Law of the Wall theory. The rendering of no-slip boundary conditions is thus modified importantly from typical practice in ocean models. The proposed boundary condition formulation is implemented in the MITgcm. Comparisons with classically formulated free-slip and no-slip cases shows the results are somewhat between the two cases, but generally closer to free-slip.

1. Introduction

Most modern ocean modeling is carried out at resolutions of $O(10\,\text{km})$ for the practical reason of computational limitations. Restricting attention to momentum, experience has shown that the so-called Reynolds averaged equations

$$\begin{aligned} u_t + uu_x + vu_y + wu_z - fv &= -p_x - \nabla\cdot\vec{F}_x, \\ v_t + uv_x + vv_y + wv_z + fu &= -p_y - \nabla\cdot\vec{F}_y, \end{aligned} \tag{1}$$

(where $\vec{F}_x, \vec{F}_y = \langle\vec{u}'u'\rangle, \langle\vec{u}'v'\rangle$ and other notation is standard) subject to a closure hypothesis on $\vec{F}_x,\ \vec{F}_y$ yield useful oceanic predictions. The form of (1) is similar to that of the Navier–Stokes equations (NSE); indeed employing

$$\vec{F}_x, \vec{F}_y = \langle\vec{u}'u'\rangle,\ \langle\vec{u}'v'\rangle = -\kappa\nabla(u,v) \tag{2}$$

with $\kappa \sim 10^{-6}\ \text{m}^2\,\text{s}^{-1}$ converts them to the NSE. On coarse oceanic grids, the quantities $\vec{F}_x, \vec{F}_y$ are usually identified with 'turbulent', or 'sub-grid scale', momentum fluxes and this implies the dynamics below the explicit model resolution must be parameterized. Although sub-grid scale parameterizations can be very detailed (e.g. bi-harmonic or Smagorinsky/Leith, see Griffies and Hallberg, 2000), an archetypical paradigm is to again employ Eq. (2), i.e., the momentum equations are written as

$$\begin{aligned} u_t + uu_x + vu_y + wu_z - fv &= -p_x + \nabla\cdot\kappa\nabla u, \\ v_t + uv_x + vv_y + wv_z + fu &= -p_y + \nabla\cdot\kappa\nabla v \end{aligned} \tag{3}$$

with κ now called the eddy viscosity. The applied κ is normally considerably larger than molecular in value, with values like $100\ \text{m}^2\,\text{s}^{-1}$ in ocean models with grid resolutions of $O(10\,\text{km})$.

Regardless of the form of the subgridscale parameterizations, specifications of the viscous interactions of the interior with the lateral boundaries are required to solve (1), and what those should be has been a matter of long debate in the modeling community. This is a particularly pronounced issue for ocean models adopting a geopotential coordinate framework, where topography is represented as a series of stair steps and conditions on tangential velocity are required at all depths. So-called 'sigma', or terrain following, coordinates avoid the issue over most of the domain, requiring only a bottom boundary viscous condition. However, they often use a minimum depth criterion, at which point the coordinates intersect a lateral boundary and tangential velocity conditions must be addressed. Finally, oceanography has a long tradition of process-oriented modeling where lateral boundaries are represented as vertical walls; the classical double gyre wind-driven circulation problem is the most well known example.

The two normally employed boundary condition forms are 'no-slip', $\vec{v}\times\vec{n} = 0$,[1] in analogy to the condition required by the NSE, and 'free-slip', characterized by $\nabla\vec{v}\cdot n = 0$. Sometimes so-called mixed boundary conditions, consisting of a linear combination of the above, are applied. It is a matter of practical modeling experience that boundary behaviors are sensitive to this choice. Given the importance of boundary currents to the global ocean, these differences routinely amplify to global sensitivities, thereby emphasizing the importance of accurately representing boundary dynamics.

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[1] The quantity $\vec{n}$ is the unit normal to the boundary.

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doi:10.1016/j.ocemod.2011.05.002

To illustrate, Wunsch, 1998 argues the net energy flux into the large-scale ocean circulation is roughly 1TW (1TW = 10^{12} W); this must in turn be balanced by a dissipation of 1TW, assuming a rough steady state. Over the ocean, the average energy loss from the balanced flow is then approximately

$$\frac{1TW}{A_{ocean}} = \frac{10^{12}\ \mathrm{W}}{3.5x10^{14}\ \mathrm{m}^2} \sim 3x10^{-3}\ \frac{\mathrm{W}}{\mathrm{m}^2}, \tag{4}$$

i.e., 3 mW m^{-2}, where A_{ocean} is the global ocean area. Local values deviate from the above mean value; differences by factors like 100 are observed (Naviera-Garabato et al., 2004). However, Eq. (4) provides a useful yardstick by which dissipation rates can be measured. From this perspective, consider the energy dissipation in a typical western boundary current of a numerical model assuming a lateral viscosity of 100 $\mathrm{m}^2\ \mathrm{s}^{-1}$, *i.e.*,

$$Diss = -\nu\rho U_x^2 h = 100\ \frac{\mathrm{m}^2}{\mathrm{s}}\,10^3\ \frac{\mathrm{kg}}{\mathrm{m}^3}\left(\frac{.5\ \mathrm{m}}{s10^4\ \mathrm{m}}\right)^2 10^3\ \mathrm{m} = .25\ \frac{\mathrm{W}}{\mathrm{m}^2}, \tag{5}$$

where h represents a typical western boundary current depth (1000 m) and a vertically averaged horizontal velocity of 0.5 ms^{-1} has been used. The no-slip condition is implicit in the estimate of the lateral shear, where the average velocity decays to zero over 10 km. The answer, .25 Wm^{-2}, is about 80 times the average rate. We don't dispute that western boundaries are areas where anomalously large dissipations might occur, but simply emphasize that if this is true, these are regions where having dissipation mechanisms motivated by physics is at a premium. Lateral eddy viscosity with no-slip is not such a parameterization.

The objective of this note is to propose a different way of implementing no-slip boundary conditions that arises from classical turbulence theory. Although the boundary is literally 'no-slip', the results for typical ocean modeling settings are very similar to 'free-slip' results. We recommend the technique to the community for further examination.

2. Theory

We accept that no-slip is the proper boundary condition to apply to the Navier–Stokes equations, as required by molecular processes. The oceanic debate about boundary conditions arises because (1) are used at scales for which molecular processes are, in an explicit sense, unimportant. The parameterizations are meant to represent momentum fluxes driven by sub-gridscale processes and the question becomes how to compute those fluxes from known model variables. Our discussion here is concerned only with the momentum fluxes imparted by the solid boundaries on the interior; fluxes elsewhere can be computed by any of the standard parameterizations. We argue that applying the no-slip condition directly to Eq. (1) is inappropriate, as literally the molecular mechanics responsible for no-slip are not represented by the parameterizations. We are not the first to argue this; it is implicit in the use of free-slip and mixed boundary conditions. Nonetheless, molecular processes will arrest any flow against a solid boundary, thereby exerting a stress on the fluid near the boundary. This argues qualitatively that free-slip boundaries are also inappropriate, inasmuch as they set boundary stresses to zero.[2]

[2] A subtle point also arises here; the no-normal flow boundary condition favors the free slip condition because the normal velocity fluctuations in Eq. (2) vanish. The occurrence of a viscous sublayer permits a transition between a turbulent outer layer, where the turbulent momentum flux in Eq. (2) is non-zero, to a wall, where normal velocities must vanish but the momentum flux does not.

Atmospheric boundary layer dynamicists have been long faced with this issue when computing the momentum transfers[3] between the lower boundary and the atmosphere, be it land or ocean. They resolve the issue by appealing to the Law of the Wall (Kraus and Businger, 1994) and to Monin–Obhukov Similarity Theory (MOST) and have found considerable success thereby (see Fairall et al., 2003 for a recent review). We emphasize the basic tenets of this approach are equally valid to the setting of a lateral boundary, thereby recommending the imposition of no-slip boundary conditions by means of a lateral drag.[4]

The idea of the Law of the Wall is that molecular processes become the dominant mechanics in a frictional sublayer at Kolmogorov-like *mm* scales. Outside this layer, viscosity quickly becomes unimportant, and control of the detailed flow is given over to a turbulence whose existence depends upon the requisite shear. Assuming equilibrium (equivalent to assuming 'slow' evolution for the free stream flow), scaling arguments lead to the governing formula

$$v_x = \frac{v_*}{\kappa_v x}, \tag{6}$$

where v is the along wall flow, $v_* = \sqrt{|\langle u'v'\rangle|}$ is the so-called 'friction velocity' (the angle brackets denote the Reynolds averaging), $\kappa_v \approx 0.4$ is the von Karman constant and x denotes lateral displacement from the boundary. Implicit in this equation is the idea that the turbulent momentum fluxes are non-divergent; the layer is often referred to as the 'constant flux' layer. The solution of (6) is

$$v = \frac{v_*}{\kappa_v}\ln\left(\frac{x}{x_o}\right), \tag{7}$$

where x_o denotes a 'roughness' length. The quantity v here represents the free-stream mean flow at location x, whose shear sustains the turbulence and the coordinate system is oriented in the free stream flow direction. Boundary layer dynamicists have determined roughness lengths x_o pertinent to a variety of land surfaces, finding x_o ranges from .001 m for ice to 1 m in extremely 'rough', rocky terrains (Lu et al., 2009). The literature on oceanic roughness lengths away from the coastal zone is not as well developed as that for the atmosphere: this point is revisited below.

We now equate the free stream velocity in (7) to that set by large scale dynamics, themselves computed for example by a 'coarse' resolution model.[5] In this case, (7) can be inverted to give the momentum flux condition experienced by the large-scale flow

$$-\vec{F}^y(x=0,y)\cdot\vec{n} = -\langle u'v'\rangle = \frac{(v\kappa_v)^2}{\left(\ln\left(\frac{\Delta}{x_o}\right)\right)^2} sgn(v), \tag{8}$$

where Δ is the location of the first velocity point of the model, the *sgn* factor appears as the model flow tangential to the boundary can be in either direction, $\vec{n}$ denotes the outward pointing normal unit vector on the boundary and we have chosen the setting of a meridional flow along a western wall. The above formula can be used to resolve the momentum flux needed by the model. We stress this boundary condition implementation is backed by decades of study of shear driven turbulence against solid objects.

Although (8) can be used directly in a model, it is standard oceanographic and meteorological practice to introduce a drag coefficient, C_d, relating the stress to the velocity at some standard distance,[6]

[3] This is also a concern for moisture and heat transfers as well.

[4] Similar implementations are implicit in the laterally averaged nonhydrostatic models of narrow coastal waterways by Stacey et al. (1995) and Bourgault and Kelley (2004).

[5] It is here that the term 'coarse' takes on quantitative meaning. Coarse resolution implies models incapable of explicitly computing the turbulent shear in the log layer.

[6] MOST is the most elaborate form of this parameterization, where the gravitational stability of the air column is considered.

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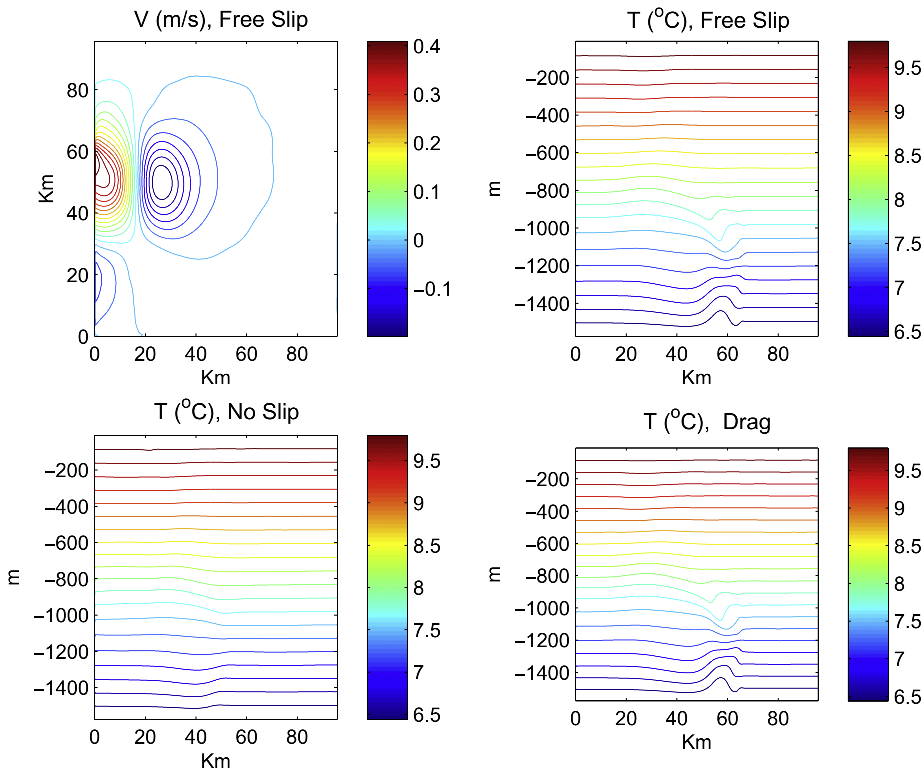

Fig. 1. Wall-vortex interactions. The nature of the interaction is indicated in the upper left panel, showing north–south velocity V in a plan view. The remaining panels are in the meridional/vertical plane at a distance of 125 m from the wall. Note the classic free-slip results resemble the present parameterization. No-slip is quite different.

$$-\langle u'v'\rangle = C_d|\vec{v}|v. \quad (9)$$

This permits the diagnosis of air-sea momentum flux from observations of a 10 m wind, for example, and here permits the diagnosis of the momentum exchange with the boundary from the model velocity at the first interior grid-point. In contrast to oceanic roughness lengths, there is a relatively well-developed ocean literature concerning relevant values for C_d. The form (9) is often used to parameterize bottom boundary layers with C_d between $(1-3)\times10^{-3}$ (Miranda et al., 1999; Arbic and Scott, 2008).

The similarity of this parameterization to that used on the bottom also connects geopotential models to terrain following models that by design have only bottom boundary layer parameterizations. Rather than handle the 'rise' components of the bottom differently that the 'run' components in a geopotential model, the present parameterization handles them in the same fashion.

3. Experiments with the MITgcm

We have examined the impact of this boundary condition implementation using the MITgcm[7] (Marshall et al., 1997) in two different settings. The first implementation is that of a heton-like vortex impinging on a north–south oriented wall. This is a setting previously described in Dewar and Hogg (2010) and the reader is referred there for details. We compare in Fig. 1 the results from three runs, one using a typical free-slip implementation, one with a classical no-slip implementation and one based on the present formulation. The value of the viscosity in all three in set to[8] 3 $m^2\,s^{-1}$. We employ a constant drag coefficient of 1.2×10^{-3} when modeling the log layer. In particular, we show the velocity field at the depth of the primary anticyclone from one of the experiments (to orient the reader in the horizontal) and the temperature field at a distance of 125 m from the wall at a time of strong wall-vortex interaction. Note that the free-slip and the present case are similar, whereas the classical no-slip case shows very muted results. The front-like temperature structure is essentially erased for the classical no-slip case, but evident in the other settings. The results have been tested at several numerical resolutions (50 m, 100, 250 m,[9] 500 m and 1 km) and viscosities and appear to be robust.

It is instructive to determine the causes of the differing results in Fig. 1. Analysis demonstrates that the boundary layer potential vorticity in the classical no-slip case is viscously dominated, i.e. the relative vorticity in the shear layer needed to meet the no-slip condition almost completely determines the potential vorticity. In contrast, in the free-slip and drag cases, potential vorticity modification in the boundary layer can occur, but does so through submesoscale and potential vorticity processes.

In Fig. 2, we show the results of the classical two gyre wind-driven circulation problem as computed by the MITgcm in a barotropic setting. This is a problem whose sensitivity to slip/no-slip/partial slip boundary conditions has been extensively studied (see Chassignet and Marshall, 2008 for a recent review). An eddy viscosity of 100 $m^2\,s^{-1}$ is employed, and a drag value of 1×10^{-3} models the log layer. We here record that the results using the present formulation lean heavily toward the free slip results, but can be seen by eye to be slightly more viscous. Chassignet and Marshall (2008) present partial slip results that also look like ours, where the model state is a function of the weighting parameter in the partial slip condition. Adcroft and Marshall (1998) point

[7] The MITgcm employs a so-called C-grid, whereas some models employ B-grids, the latter differing from the former only in the placement of the velocity grids. As such momentum fluxes are needed at similar location on the two grids. The present ideas should apply to both.

[8] Eddy viscosity is used to parameterize momentum fluxes away from the boundary when the present side-drag formulation is used.

[9] The experiments in Figs. 1-3 are at a horizontal resolution of 250 m and a vertical resolution of 16 m.

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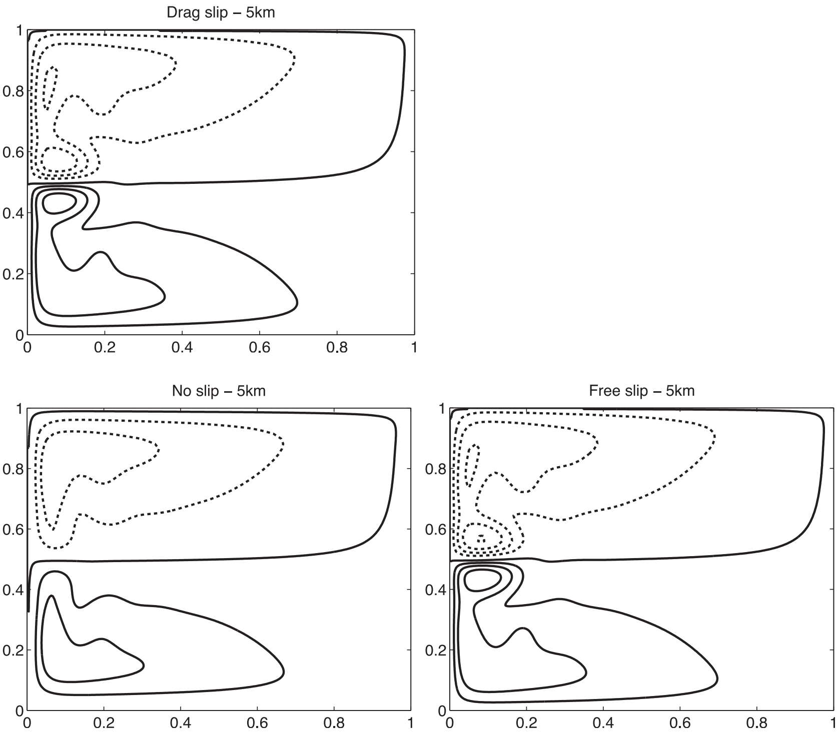

Fig. 2. The classic, barotropic wind-driven double gyre problem. The plots are of free surface height (*CI* = .08 m). The top panel is the present parameterization, the bottom left is no-slip and the bottom right is free slip. A constant drag value of 1×10^{-3} models the log layer and the basin is 1200 km $\times$ 1200 km. The results of the side drag are much closer to the free-slip results although indications of a slightly more viscous evolution are evident to the eye.

out additional difficulties with boundary parameterizations in ocean models with $O(10\text{ km})$ resolution when the coastlines slant.

These examples are at extremes of 'coarse' resolution modeling. Both are 'coarse' relative to the scale of the turbulent shear sublayer, but the first is a model capable of resolving the submesoscale (Molemaker et al., 2005) while the latter at most can compute mesoscale. A critical and unresolved question is the extent to which the present parameterization survives the great scale separation inherent in the Fig. 2. We speculate it will be necessary to supplant the side-drag parameterization with an intermediate parameterization of the submesoscale.[10] This is motivated by, among other considerations, the fact that free-slip solutions tend to be weakly dissipative in western boundary layers, and can have overly exaggerated jet extensions. Further studies of the highly resolved kind shown in Fig. 1 are needed to help address this issue, with the advantage being that the drag conditions are motivated by physics, and do not damp the submesoscale, dynamical response of the model.

4. Summary

We argue here that classical Law of the Wall theory applies to the problem of lateral boundary viscous interaction between the flows computed by most 'coarse' resolution ocean models and walls. The results of implementing such a no-slip boundary condition are shown in example cases to resemble much more closely those from free-slip ocean models than classical no-slip. It is still true that the parameterization computes a momentum flux into the wall and thus is more 'viscous' than the classical free-slip implementation. So-called mixed boundary conditions are also often used in ocean modeling, although the weighting between the velocity and its normal gradient is a free parameter about which little is known. To the extent parallels exist between the present side drag parameterization and mixed boundary conditions, boundary roughness lengths correspond to the weighting parameters and can perhaps be used to constrain them. In any case, the Law of the Wall is physically motivated and well grounded in the fluid mechanical literature and this is desirable relative to the ad hoc choices often made for mixed condition parameters or the suspect application of free slip/no slip boundary conditions. Clearly there are many avenues for exploration in establishing or refuting the validity of the side drag boundary condition. We suggest that this lateral boundary condition formulation be subject to more extensive testing and scrutiny by the oceanography modeling community.

[10] An interesting possibility for such a parameterization is found in Mariano et al. (2003).

Acknowledgements

This work was supported by NSF grants OCE-0960500, 0961485 and 1049131, the latter in support of the multi-agency Earth Systems Modeling effort. Useful conversations with Georges Weatherly are gratefully acknowledged. Suggestions by Alistair Adcroft and an anonymous reviewer were central to improving the initial submission.

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